Problem collection

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When I was in secondary school, I used to participate extensively in academic contests in mathematics, informatics and physics. Those competitions (particularly the mathematics and informatics ones; the physics ones are somewhat different and I did not enjoy them as much) are based on solving elementary, easy-to-state problems whose solution nevertheless may involve a great deal of ingenuity.

After entering university, I started to think about creating my own problems. At the beginning, it was a very difficult endeavor: it could not occur to me how to come up with original ideas that would make for interesting problems. However, over time I've become more proficient in this, and many problems designed by me have appeared in different competitions, including prestigious international ones. I have come to enjoy very much this process of creating problems.

Here is a collection of all the problems I have proposed for different mathematics and informatics competitions, with personal commentary. All the problems are authored by me except in some few cases in which I acknowledge a coauthor in the commentary. In some cases I was informed after the contest (or before the contest, for some less important contests in which unoriginal problems are acceptable) that the problem had already appeared; this is acknowledged in the commentary. All other problems are completely original to the best of my knowledge. Comments and feedback are welcome.

Depending on how much I like the problems, they are rated with zero, one, two, or three stars. Problems with zero stars are either probably not worth the reader's time or the ideas present in them are not original at all.

International Olympiad in Informatics

Thounsands Islands (2022 Day 2 `islands`) (⋆⋆⋆) : You are given a labelled directed graph. Initially, a token is on vertex 0. A *move* consists in moving the token from a vertex u to a vertex v so that there exists an edge $u \to v$ and reversing the direction of the edge traversed so that it is now $v \to u$. A *tour* is a nonempty sequence of moves such that the token starts and ends in vertex 0, there are no two consecutive moves which use the same edge, and the final graph is the same as the original graph (that is, the orientation of the edges is the same, so each edge has been traversed an even number of times).

Design a linear-time algorithm which constructs a tour if there is one or concludes that it does not exist.

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Codeforces Rounds

Good Pairs (CodeTON Round #1 A) ($\star\star$) : You are given an array $a_1, a_2, \ldots, a_n$ of positive integers. A *good pair* is a pair of indices (i, j) with $1 \leq i, j \leq n$ such that, for all $1 \leq k \leq n$, the following equality holds:

$$|a_i - a_k| + |a_k - a_j| = |a_i - a_j|,$$

where $|x|$ denotes the absolute value of x.

Find a good pair. Note that i can be equal to j. ($n \leq 10^5$).

Click to toggle commentary

Subtract Operation (CodeTON Round #1 B) ($\star\star$) : You are given a list of $n \leq 2 \cdot 10^5$ integers. You can perform the following operation: you choose an element x from the list, erase x from the list, and subtract the value of x from all the remaining elements. Thus, in one operation, the length of the list is decreased by exactly 1.

Given an integer k ($k > 0$), find if there is some sequence of $n - 1$ operations such that, after applying the operations, the only remaining element of the list is equal to k.

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Make Equal With Mod (CodeTON Round #1 C) ($\star$) : You are given an array of $n \leq 10^5$ non-negative integers $a_1, a_2, \ldots, a_n$. You can make the following operation: choose an integer $x \geq 2$ and replace each number of the array by the remainder when dividing that number by x, that is, for all $1 \leq i \leq n$ set a_i to a_i mod x.

Determine if it is possible to make all the elements of the array equal by applying the operation zero or more times.

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K-Good (CodeTON Round #1 D) : We say that a positive integer n is k-good for some positive integer k if n can be expressed as a sum of k positive integers which give k distinct remainders when divided by k.

Given a positive integer $n \leq 10^{18}$, find some $k \geq 2$ so that n is k-good or tell that such a k does not exist.

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Equal Tree Sums (CodeTON Round #1 E) ($\star\star$) : You are given an undirected unrooted tree, i.e. a connected undirected graph without cycles.

You must assign a **nonzero** integer weight to each vertex so that the following is satisfied: if any vertex of the tree is removed, then each of the remaining connected components has the same sum of weights in its vertices. $n \leq 10^5$.

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Parametric MST (CodeTON Round #1 F) : You are given $n \le 2\cdot 10^5$ integers $a_1, a_2, \ldots, a_n$, $-10^6 \le a_i \le 10^6$. For any real number t, consider the complete weighted graph on n vertices $K_n(t)$ with weight of the edge between vertices i and j equal to $w_{ij}(t) = a_i \cdot a_j + t \cdot (a_i + a_j)$.

Let $f(t)$ be the cost of the minimum spanning tree of $K_n(t)$. Determine whether $f(t)$ is bounded above and, if so, output the maximum value it attains.

Click to toggle commentary

Cycle Palindrome (CodeTON Round #1 G) ($\star\star$) : We say that a sequence of n integers $a_1, a_2, \ldots, a_n$ is a palindrome if for all $1 \le i \le n$, $a_i = a_{n-i+1}$. You are given a sequence of $n \le 2 \cdot 10^5$ integers $a_1, a_2, \ldots, a_n$ and you have to find, if it exists, a *cycle permutation* σ so that the sequence $a_{\sigma(1)}, a_{\sigma(2)}, \ldots, a_{\sigma(n)}$ is a palindrome.

A permutation of $1, 2, \ldots, n$ is a bijective function from $\{1, 2, \ldots, n\}$ to $\{1, 2, \ldots, n\}$. We say that a permutation σ is a cycle permutation if $1, \sigma(1), \sigma^2(1), \ldots, \sigma^{n-1}(1)$ are pairwise different numbers. Here $\sigma^m(1)$ denotes $\underbrace{\sigma(\sigma(\ldots\sigma}_{m \text{ times}}(1)\ldots))$.

Click to toggle commentary

Equal LCM Subsets (CodeTON Round #1 H) : You are given two sets of positive integers A and B. You have to find two non-empty subsets $S_A \subseteq A$, $S_B \subseteq B$ so that the least common multiple (LCM) of the elements of S_A is equal to the least common multiple (LCM) of the elements of S_B. $|A|, |B| \le 1000, a_i, b_i \le 4 \cdot 10^{36}$.

Click to toggle commentary

Neighbor Ordering (CodeTON Round #1 I) : Given an undirected graph G, we say that a *neighbour ordering* is an ordered list of all the neighbours of a vertex for each of the vertices of G. Consider a given neighbour ordering of G and three vertices u, v and w, such that v is a neighbor of u and w. We write $u <_v w$ if u comes after w in v's neighbor list.

A neighbour ordering is said to be *good* if, for each simple cycle $v_1, v_2, \ldots, v_c$ of the graph, one of the following is satisfied:

- $v_1 <_{v_2} v_3, v_2 <_{v_3} v_4, \ldots, v_{c-2} <_{v_{c-1}} v_c, v_{c-1} <_{v_c} v_1, v_c <_{v_1} v_2$.
- $v_1 >_{v_2} v_3, v_2 >_{v_3} v_4, \ldots, v_{c-2} >_{v_{c-1}} v_c, v_{c-1} >_{v_c} v_1, v_c >_{v_1} v_2$.

Given a graph G ($n, m \le 5 \cdot 10^5$), determine whether there exists a good neighbour ordering for it and construct one if it does.

Click to toggle commentary

Spain Olympiad in Informatics

Baq and the distances between cities (Final 2020, Day 2) : Given n, construct a weighted complete graph on n vertices so that the weights of the edges are a permutation of $\binom{1,\dots,}{n2}$ and the distance between any two points is equal to the weight of the edge between them.
Click to toggle commentary

Expensive transport (Final 2020, Day 2) : You are given a weighted graph ($n, m \leq 1000$). Find the minimum cost to go from s to t if the cost of traversing an edge is the sum of the weights of all the previously traversed edges.
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Game on a graph (Final 2020, Day 2) ($\star$) : A and B play a game on a labelled graph G ($n, m \leq 10^5$). At the start of the game, one vertex is marked as the "winning vertex". Players take turns, with A starting. At each turn, the player chooses one connected component of the graph and deletes the vertex with the smallest number (label) in the component. The player who deletes the winning vertex wins. Find for which winning vertices player A has a winning strategy.
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Hola (First qualifier 2021) : Determine whether a string is a permutation of "`hola`".
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Closed under subtraction (First qualifier 2021) : A set S of integers is said to be closed under substraction if for all $a, b \in S$, $a \neq b$ either $(a - b) \in S$ or $(b - a) \in S$. You are given a set of $n \leq 5 \cdot 10^5$ integers, determine whether it is closed under subtraction.
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Squares in the notebook (First qualifier 2021) : There are $m \leq 10^9$ horizontal lines with y coordinates $1, 2, \dots, m$ and $n \leq 10^6$ vertical lines with x coordinates $x_1, \dots, x_n$. Compute the number of squres with vertices in the intersections of those lines.
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Beautiful grid (First qualifier 2021) ($\star$) : You are given two integers n, m. Color each cell of a $n \times m$ grid black or white such that the number of black cells is equal to the number of white cells and the number of adjacent pairs of different colors is equal to the number of adjacent pairs of equal colors or determine that it is impossible.
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Components of the grill (First qualifier 2021) ($\star$) : You are given $n \leq 10^6$ positive integers $a_1, \dots, a_n$ and $m \leq 10^6$ integers $b_1, \dots, b_m$. Find the number of (topologically) connected components of the union of the segments with endpoints $(0, i), (a_i, i)$ for $1 \leq i \leq n$ and endpoints $(j, 0), (j, b_j)$ for $1 \leq j \leq m$.
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Pairs (Second qualifier 2021) : You are given a list of $2n \leq 10^5$ integers, determine whether it is possible to group them in pairs of equal sum.

Click to toggle commentary

Numbers on a grid (Second qualifier 2021) ($\star$) : You are given an $n \times m$ grid ($n, m \leq 10^6$) and in each cell initially the number 0 is written. You are given $q \leq 10^6$ queries, in each query you have to replace all the numbers in a given row or column by a given number x_i. Find the sum of all the numbers in the grid after all the queries.

Click to toggle commentary

Guess the Factorial (Final 2021, Day 1) ($\star$) : There is a hidden integer $1 \leq n \leq 40\,000$ that you have to guess. You may ask questions of the form "Is $n! + a$ divisible by b?" where $1 \leq a, b \leq 10^9$. Find n in few queries (60 for full score, up to 100 for partial score).

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Merlion Statues (Final 2021, Day 1) ($\star\star\star$) : Steven is walking around Singapore and has found a runway which has Merlion-shaped fountains at its sides.

Steven has decided to walk through the runway, and while he is walking through it each time he passes along a statue he writes down in his notebook at which side he sees the merlion statue: at his right or at his left. Steven starts walking forward, but he can choose to turn around at any point in the walk, and he can turn around multiple times. His walk has to finish at the same place it started: the entrance of the runway.

Now Steven wants to remember how the runway was and how was the walk he did, but he only has the annotations. From these annotations, you must reconstruct a possible distribution of statues and a possible walk that generates the annotations. It is possible that the annotations given in the input are wrong and it is impossible to construct a walk generating them, if so you must indicate it.

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Bicoloration with Distances (Final 2021, Day 1) ($\star$) : You are given a weighted graph ($n, m \leq 10^5$). Determine the maximum value of D so that the graph can be 2-colored so that every two vertices at distance less than D are of different colors.

Click to toggle commentary

Dreamcatcher (Final 2021, Day 2) ($\star\star$) : Given two integers n, k, you must construct a permutation of $p_1, \ldots, p_n$ $1, 2, \ldots, n$ so that if one has n points in a circle labelled consecutively $1, 2, \ldots, n$ and one draws the segments joining p_i and p_{i+1} for $1 \leq i \leq n-1$ then the segments have exactly k intersections in the interior of the circle.

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Minimum XOR (Final 2021, Day 2) ($\star\star\star$) : There is a set S of n nonnegative integers you have to determine. You can make the following query: you send a nonnegative intger y, and you get as a result $\min_{x \in S}(x \oplus y)$, where $x \oplus y$ denotes the bitwise XOR operation. Determine the set using at most $2n$ queries.

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Distributing Weapons (Final 2021, Day 2) ($\star$) : You are given a rooted tree ($n \leq 10^6$) in which each vertex has at most 2 children. Compute the maximum number of nodes that can be painted so that for every node with 2 children, the number of painted vertices in the left subtree is equal to the number of painted vertices in the right subtree.
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Knights and Knaves (Final 2021, Day 2) ($\star$) : There is a row with $n \leq 10^9$ people. Each person in the row is either a knight or a knave. Knights always tell the truth and knaves always lie.

You can ask one person in the row "In which direction is the nearest knight in the row?". If you ask a knight, it will answer the direction in which the nearest knight in the row (other than him) is. If you ask a knave, it will answer the opposite direction of the direction in which you can find the nearest knight in the row. It is guaranteed that the distribution of knights and knaves in the row will be such that the answers are uniquely determined, that is, there won't exist any knight or knave such that the nearest knights in both directions are at the same distance.
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Divisibility (Qualifier 2022) ($\star$) : You are given two distinct integers $a, b \leq 10^9$. Find an integer $1 \leq c \leq 2 \cdot 10^9$ so that $a + b + c | a \cdot b \cdot c$. Solve $T \leq 10^5$ testcases.
Click to toggle commentary

VonitAs Sequences 2 (Qualifier 2022) : You are given a sequence $a_0, \ldots, a_{n-1}$ ($n \leq 2 \cdot 10^5$) and you can make the following operation: for some integers k and i, you can add k to either the i first elements of the sequence or the i last ones. Determine the minimum number of operations to make the sequence unimodal (increasing and then decreasing, or decreasing and then increasing).
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Diagonals of the grid (Qualifier 2022) ($\star$) : In an $n \times m$ ($n, m \leq 500$) grid each square can contain a diagonal line connecting two opposite vertices. Some squares are blocked and do not contain any segment. We say that a vertex is active if there is a diagonal that connects it with another vertex. We want all the active vertices of the grid to be connected, in other words, we want to be able to go from one another moving along the diagonals. To achieve this goal we can swap the diagonal of a square with the other diagonal (the one that connects the other two vertices). Given a grid, you are asked to calculate the minimum number of diagonal turns that must be made to get the active vertices connected. If there is no solution, you must indicate that this is the case.
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Pair (Qualifier 2022) : Given a number $n \leq 10^4$, the judge is keeping hidden two different positive integers $1 \leq a < b \leq n$ that you must guess. To do this, you can ask the following type of questions: you divide the numbers from 1 to n into two groups and ask whether a and b are in the same group or in different groups. You must determine a and b by asking at most 30 questions.
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Windmills (Qualifier 2022) : There are $n \leq 10^5$ points in the plane. From each point comes a ray originally in the direction of the negative Y axis. The rays start to rotate counterclockwise all at the same speed. A point (and its associated ray) is eliminated when a ray from another point passes through it. Print the order in which the points are eliminated.

Click to toggle commentary

Painting with Distances (Final 2022, Day 1) ($\star\star$) : Manuela really likes to color graphs. She has decided that she wants to color connected graphs with exactly n ($n \leq 100$) vertices. To color this type of graph she has made a plan consisting of m ($m \leq 100$) integers $a_1, a_2, \ldots, a_m$. Manuela colors the graph in the following way: in day 0 she selects a vertex and paints it. In the day i (for each i between 1 and m) she paints all the vertices of the graph that are exactly a_i away from any vertex already painted. (The distance between two vertices is the number of edges on the shortest path between them). Manuela says that a plan is *cool* if, for any connected graph of n vertices, a vertex can be selected in day 0 that makes all vertices be painted in the end. Given n and a plan, determine whether the plan is cool and, if it is not, provide an example of a graph with n vertices and connected such that, regardless of which vertex is chosen in the first day, there is always one vertex left unpainted.

Click to toggle commentary

Interactive GCD (Final 2022, Day 1) : There are two hidden integers $1 \leq x, y \leq 10^{18}$ that you must guess. In order to do that, you can ask questions where you give two integers a, b and you receive the value of $\gcd(|x-a|, |y-b|)$. Guess the integers using few queries.

Click to toggle commentary

Sums (Final 2022, Day 1) : Berta and Blanca have n cards, each with a positive integer b_i. Berta proposes a game: she will choose a positive integer m and then Blanca will have to deal the n cards in separate piles; after that, Berta will choose two cards from the same pile: if the sum of the values b_i and b_j of the cards she has chosen is divisible by m Berta will win, if it is not divisible by m Blanca will win. The piles must contain at least 2 cards and there can be only one pile with n cards. Blanca wants to know if she can deal the cards in piles in such a way that it is impossible for Berta to win and, if it is possible, she wants to do it with the minimum number of piles possible. Given $n \leq 10^5$, $m \leq 10^9$ and the n values b_i of the cards determine whether Blanca can win and the minimum number of piles she needs to do so.

Click to toggle commentary

Sequence of Next Multiples (Final 2022, Day 2) ($\star$) : Given a positive integer m, define the sequence $a_1 = m$ and a_i for $i \geq 2$ is the smallest multiple of i greater than or equal to a_{i-1}. If the resulting sequence has no two consecutive equal terms, we say that m is special. Find the n-th ($n \leq 10^7$) special number.

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Unsorted (Final 2022, Day 2) : Given a list $a = (a_1, \ldots, a_n)$ of n distinct integers, we say that a reordering (or permutation) $b = (b_1, \ldots, b_n)$ of the list a is *completely unsorted* if every contiguous nonempty sublist of b is not equal to the sublist with the same indices of a but ordered. That is, if for all $1 \leq i \leq j \leq n$, the sequence of numbers $b_i, b_{i+1}, \ldots, b_j$ is not equal to the sequence of numbers $a_i, a_{i+1}, \ldots, a_j$ but ordered. Given a list, print a completely unsorted reordering.

Click to toggle commentary

Eating cookies (Final 2022, Day 2) ($\star$) : Juan has $n \leq 1000$ different kinds of cookies. Of the i-th kind he has a_i cookies. John wants to eat all the cookies, but he must follow a constraint: each day there must be a type of cookie t such that the number of cookies he eats of type t that day must equal the total number of cookies he eats of types other than t that day. Can Juan eat all the cookies? Design an "eating schedule" if it is possible.

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Adding to a Composite Number (Qualifier 2023) : Given $n \leq 1000$, you are asked to print a sequence of n positive integers $a_1, \ldots, a_n$, $a_i \leq 5000$ that satisfies the following two conditions:

- For all $1 \leq i < j \leq n$, $\sum_{k=i}^{j} a_k = a_i + \ldots + a_j$ is a composite number.
- For all $1 \leq i < n$, the numbers a_i and a_{i+1} are coprime, i.e., $\gcd(a_i, a_{i+1}) = 1$.

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Flipping (Qualifier 2023) ($\star$) :

Let n be a given positive integer. The judge has a hidden number $1 \leq x \leq n$ that you must determine. To do this, you can ask a series of questions.

There is a permutation $p_1, \ldots, p_n$ where initially $p_i = i$ for all $1 \leq i \leq n$. You can ask the following query: you give an index i and the judge answers if $p_i < x$, $p_i = x$ or $p_i > x$. But after each query, if j is the index such that $p_j = x$, you flip the $[i, j]$ segment (if $i \leq j$) or the $[j, i]$ segment (if $i > j$) of the permutation, ie, denoting by p' the new permutation after the question, you have that $p'_i = p_j, p'_{i+1} = p_{j-1}, \ldots, p'_j = p_i$ if $i \leq j$ and analogously $p'_j = p_i, p'_{j+1} = p_{i-1}, \ldots, p'_i = p_j$ if $i > j$.

Your goal is to determine the number x without asking too many questions.

Click to toggle commentary

Min times max (Qualifier 2023) : Given $n \leq 10^6$ integers, you must calculate:

$$\sum_{i=1}^{n} \sum_{j=i}^{n} \sum_{k=j+1}^{n} \sum_{l=k}^{n} \min(a_i, \ldots, a_j) \times \max(a_k, \ldots, a_l) \pmod{10^9 + 7}$$

Click to toggle commentary

Distance to Multiple or Divisor (Final 2023, Day 1) : The judge has a hidden number $n \leq 10^{18}$ that you must determine. To do this, you can ask questions of the following two types:

1. You provide an integer m and the judge answers "Sí" if m is a multiple of n **o** if m is a divisor of n, and "No" otherwise. This question has cost 1.

2. You provide an integer m and the judge answers you an integer d which is the minimum distance from m to any of the multiples or divisors of n, i.e., $d = \min_{x \in D_n \cup M_n} |m - x|$, where D_n is the set of divisors of n and M_n is the set of multiples of n. The i-th time you ask this question, the question has cost i^2.

Your goal is to determine the number n without the sum of costs of your questions becoming too large.

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Szegedin (Final 2023, Day 1) ($\star$) : We say that a pair of integers (a, b) is *szegedin* if there exist positive integers u, v, x, y such that:

- $a = u + v$.
- $b = x + y$.
- u and v are divisors of b.
- x and y are divisors of a.

Given $T \leq 10^5$ pairs (a, b) with $a, b \leq 10^{18}$, determine whether they form a segedinan pair.

Click to toggle commentary

Whispering (Final 2023, Day 2) ($\star$) : Given a graph G ($|V|, |E| \leq 10^5$) and two vertices A, B, find a path P with minimum *score* between A and B, where the score of a path P is the maximum of the scores of the vertices of G with respect to path P, and the score of a vertex v with respect to path P is the number of vertices in its neighborhood (including v) that are part of path P.

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Goulash (Final 2023, Day 2) : Given $A, B, C, N \leq 10^8$, determine if there exists integers $0 \leq x \leq A, 0 \leq y \leq B, 0 \leq z \leq C$ so that $4x + 5y + 6z = N$.

Click to toggle commentary

Valuable List (Final 2023, Day 2) ($\star$) : Given $v \leq 10^6$ and $k \leq 10^{15}$, find the k-th list of integers in lexicographic order with $v = n + \sum_{i=1}^{n-1} |a_i - a_{i+1}|$, where n is the length of the list.

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Petrozavodsk Training Camps

Beautiful permutation (Ptz UPC Contest 2021) ($\star$) : A permutation $a_0, a_1, \ldots, a_{n-1}$ of $0, 1, \ldots, n-1$ is said to be *beautiful* if the sequence $b_0, \ldots, b_{n-1}$ defined as $b_i = |a_i - i|$ is also a permutation of $0, \ldots, n-1$.

Given $n \leq 10^6$, construct a beautiful permutation of n elements or determine that it does not exist.

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Cartesian MST (Ptz UPC Contest 2021) ($\star$) : Let G and H be two weighted undirected simple graphs. We define the *cartesian product* of the two graphs, $G \square H$, as the graph whose vertex set is the cartesian set product of the vertex sets of the two graphs $V(G) \times V(H)$ and in which there is an edge between vertices (u_1, v_1) and (u_2, v_2) if and only if:

- $v_1 = v_2$ and there is an edge (u_1, u_2) in G. In this case, the edge$((u_1, v_1), (u_2, v_2))$ in $G \square H$ has the same weight as the edge (u_1, u_2) in G.
- or $u_1 = u_2$ and there is an edge (v_1, v_2) in H. In this case, the edge$((u_1, v_1), (u_2, v_2))$ in $G \square H$ has the same weight as the edge (v_1, v_2) in H.

You are given two connected graphs G and H. Compute the total weight of the minimum spanning tree of $G \square H$. The graphs are of sizes $\leq 10^5$.

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Display of Springs (Ptz UPC Contest 2021) :

Elater is a great magician. His most famous show is "Elater's Super Spectacular Display of Springs". The show consists in the following:

There are n elastic springs in a line attached to the ceiling. Spring i is attached at height h_i and has a *stiffness constant* k_i. If we attach a weight of mass w to the lower end of the i-th spring, the weight will descend to a height H given by the formula:

$$H = h_i - \frac{w}{k_i}.$$

Elater will take questions from people in the audience. When asked about a positive integer w, Elater will be able to pick the spring which, if a weight of mass w is attached to its lower end, will descend to a height lower (closer to the floor) than all the other springs (in case of a tie, he will be able to pick one of the springs with the lowest descend height). To accomplish such an amazing feat, Elater has the help of his dear assistant, Hooke.

Before the show, Hooke has some time to do measurements on the springs. He can not directly measure the values of h_i and k_i, but he can choose two springs a and b and a mass w, try attaching a weight of mass w to the two springs, and see which of them goes closer to the floor with that weight. Before the show, he has time to do up to 20 000 such measurements. Also, after each question, Elater can distract the audience for a while, so that Hooke can do up to 20 more measurements before inconspicuously whispering the answer to Elater.

Can you help Hooke make the show successful? Assume that the springs are attached high enough, and the masses are small enough, so that the weights never reach the floor during any possible measurement. Also note that the weights are removed after each measurement.

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Friendship Circles (Ptz UPC Contest 2021) : Let $p_0, p_1, \ldots, p_{n-1}$ be $n \leq 10^5$ points in the plane in general position. We say that two points are *friends* if one can draw a circle that contains both points in its interior and all the other $n-2$ points in its exterior. Print the indices of the points that are friends with p_0.

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Königsberg Bridges (Ptz UPC Contest 2021) : Given a graph, we say it is *Königsbergsy* if there is a simple path that goes through all of its bridges. Here, a *bridge* is an edge that disconnects the graph when removed. And recall that a simple path is a path that visits each vertex at most once.

Given a graph G (size $\leq 10^5$), we want to add some edges to it to make it Königsbergsy. (You may add more than one edge between the same pair of vertices). Determine the maximum number of bridges that the resulting graph can have.

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UPC Contest

Fibonacci-like sequences (Final 2022) : Inspired by the Fibonacci sequence:

$$F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2} \text{ for } n \geq 2$$

Xavier defined his own sequence of numbers:

$$X_0 = 0, X_1 = 1, X_n = X_{X_{n-1}} + X_{X_{n-2}} \text{ for } n \geq 2$$

Max also wants to have his own sequence of numbers, so he modified Xavier's definition a bit:

$$M_0 = 1, M_1 = 0, M_n = M_{M_{n-1}} + M_{M_{n-2}} \text{ for } n \geq 2$$

Given an integer $0 \leq n \leq 10^9$ and a character `X` or `M`, output the n-th term of the corresponding sequence.

Click to toggle commentary

Equal subset products (Final 2022) : You are given n rational numbers $\frac{a_1}{b_1}, \ldots, \frac{a_n}{b_n}$. Find two distinct (not necessarily nonempty) subsets $I, J \subseteq [n]$ such that:

$$\prod_{i \in I} \frac{a_i}{b_i} = \prod_{j \in J} \frac{a_j}{b_j}$$

Constraints: $n \leq 10^5$, $1 \leq a_i, b_i \leq n$

Click to toggle commentary

Catalan Olympiad in Informatics

Eating Chocolate (Qualifier 2020) : Pau has a chocolate bar of $n \times m$ ($n, m \leq 10^5$) pieces and wants to eat it all. In order to do that, he will repeatedly select one row or column and eat all the remaining pieces of that row or column. Each row and each column has a *pleasure multiplier*, indicating the number of pleasure units that Pau will obtain for each piece he eats if he selected that row or column. Compute the maximum amount of pleasure that Pau can achieve.

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OIcat in teams (Final 2020) ($\star$) : You are given an integer $n \leq 10^5$ and three permutations p, q, r of $1, \ldots, n$. Find whether there exist three different indices i, j, k so that $p_i = \max\{p_i, p_j, p_k\}$, $q_j = \max\{q_i, q_j, q_k\}$, and $r_k = \max\{r_i, r_j, r_k\}$.

Click to toggle commentary

Almost palindrome (Final 2021) : A string s is an *almost palindrome* if two characters can be swapped to make a palindrome. Determine if s ($|s| \leq 10^6$) is an almost palindrome.

Click to toggle commentary

Spain Girls' Olympiad in Informatics

Little Red Riding Hood (Final 2022, Day 1) : Little Red Riding Hood is going to her grandma's house through the forest, which is modelled as a weighted graph ($n, m \leq 10^5$). There is a unique shortest path from her grandma's house to any other vertex in the graph. LRRH will always pick the edge going towards her grandma's house in the shortest path, but the wolf can trick LRRH up to $k \leq 20$ times to pick a different edge instead. Find the maximum possible distance travelled by LRRH.

Click to toggle commentary

Consecutive Product (Final 2022, Day 2) : You are given two integers $1 \leq n, m \leq 10^9$, find the greatest k such that there exists k consecutive integers so that their product is not divisible by n^m.

Click to toggle commentary

Expansion (Final 2022, Day 2) : Given a graph, its *expansion* is computed in the following way: each of its edges is substituted by a vertex with two edges to the endpoints of the former edge. You are given a simple graph G ($n, m \leq 10^5$), determine if it is the expansion of another graph.

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FME Contest

UPC-3 (2020) : Find two two-digit integers x, y such that $x+y = 10(x-y)$ and $x-y = \tau(x)$.
Click to toggle commentary

Enthusiasm (2020) : A number if said to be *enthusiastic* if it can be expressed as a product of factorials. Determine whether an integer $\leq 10^9$ is enthusiastic.
Click to toggle commentary

Closest leaves (2020) : Find the distance between the two closest leaves in a tree.
Click to toggle commentary

Odious (2022) : A number is *odious* if it has an odd number of ones in its binary representation. Find the first threee consecutive odious numbers whose sum is a power of two.
Click to toggle commentary